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EXPLORING EDUCATIONAL EFFICIENCY DIVERGENCES ACROSS SPANISH REGIONS IN PISA 2006*

JOSÉ MANUEL CORDERO FERRERA
EVA CRESPO CEBADA
FRANCISCO PEDRAJA CHAPARRO
University of Extremadura

DANIEL SANTÍN GONZÁLEZ
Complutense University of Madrid

Recently published results from the PISA 2006 Report show the existence of significant differences in performance scores among students from different Spanish regions participating in this evaluation. The aim of this paper is to use the information provided by this dataset in order to identify the causes of those divergences after controlling for educational inputs and environmental variables. For this purpose, we explicitly consider that education is a multi-input multi-output production process subject to inefficient behaviours, which can be identified at student level using a parametric stochastic distance function approach. Our findings suggest that La Rioja and Castile-Leon are the most efficient regions in Spain while Andalusia, Catalonia and the group composed of the regions that do not participate in the PISA with an extended sample are the worst. In addition, we conclude that most divergences in efficiency are attributable to students who are immigrants or repeating some course. In contrast, other factors such as class size or the type of school ownership do not seem to account for differences in students' performance.

Key words: Efficiency, Education, parametric distance function.

JEL classification: C14, H52, I21.

One of the main goals in the field of the economics of education is to define the relationship between school inputs, student background and achievement at school. However, after five decades of research, the evidence found is still not solid enough, especially regarding the role of school inputs [Cohn and Geske (1990), Hedges *et al*. (1994), Hanushek (1997, 2003)]. This implies a serious

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drawback for policy-makers taking decisions about the allocation of public resources devoted to enhancing the accumulation of human quality.

What we actually know is that education is a highly complex process with variables such as organization or non-monetary inputs implied in production [Vandenberghe (1999)], which make it extraordinarily difficult to define a general educational production function that accurately includes all the relevant factors. Furthermore, it should be taken into account that there may be inefficient behaviours in the learning process which may be due to multiple reasons such as the way in which resources are organized and managed, the motivation of the agents involved in the process or the structure of the educational system itself [Nechyva (2000), Woessman (2001)].

In order to tackle the efficiency issue in education, many studies use deterministic nonparametric data envelopment analysis in empirical evaluations. Pioneer studies applying data envelopment analysis in education originate with Bessent and Bessent (1980), Charnes, Cooper and Rhodes (1981) and Bessent *et al*. (1982)[1]. Other studies have considered parametric methodologies, mainly using the Cobb-Douglas specifications, but also the translog functional form proposed by Christensen, Jorgenson and Lau (1971). These studies have included Jiménez (1986), Callan and Santerre (1990), Gyimah-Brempong and Gyapong (1992), Deller and Rudnicki (1993), Grosskopf *et al*. (1997) and Perelman and Santín (2008). The main advantage of the parametric translog function is its highly flexible nature, which allows the study of second order interactions in the production process as well as allowing the calculus of output-input partial derivatives. Nevertheless it is worth noting that most of the applied work developed around this issue is conducted using the school as the Decision Making Unit (DMU). However, Summers and Wolfe (1977) and Figlio (1999) used student-level data in their econometric studies; both concluded that the student level is more appropriate than higher levels of aggregation. Their findings show that school inputs matter but that their impact on different types of student varies considerably. In addition to this, Hanushek, Rivkin and Taylor (1996) conclude that, in the econometric estimation of the educational production function, data aggregation at school, district or even country level implies an upwards bias of estimated school resource effects.

In this paper we propose the use of a parametric stochastic distance function at student level. Under this specification, we explicitly consider that education is a process in which students use their own and school inputs in order to transform them into academic results, subject to inefficient behaviours that can be identified at both student and school levels. Moreover, parametric stochastic distance functions allow us to deal simultaneously with multiple outputs (e.g. math, reading and science test scores) and multiple inputs (including school inputs, student background and peer-group characteristics) within a stochastic framework. We adopt a translog specification to estimate the parametric stochastic distance function at the student level. This allows us to calculate several aspects of educational technology, mainly output elasticities with respect to inputs and outputs. More-

(1) For an empirical survey of frontier efficiency techniques in education, see Worthington (2001).

over, we employ the methodology proposed by Battese and Coelli (1995) to find out the main driving factors for explaining educational inefficiency.

In order to illustrate the potentialities of the approach proposed here, we provide an application to Spanish educational data from the Programme for International Student Assessment (PISA), implemented in 2006 by the Organization for Economic Cooperation and Development (OECD). Through this initiative, the cognitive skills of students around the world are measured with the aim of identifying potential causes of school failure and serving as a basis for educational policy. The study was first developed in 2000 and it has been carried out every three years with a regular increase in the number of participating schools and countries. The PISA 2006 data base comprises information about over 400,000 students belonging to 57 countries of which 30 countries belong to the OECD.

This database includes a wide variety of background information on the students collected through individual questionnaires. Most of this information refers to the students' family background and learning strategies. In addition, the study also conducted interviews with the principals of the respective schools in order to collect information on the school resources, the number of teachers in the school, the responsibility of the school regarding school relevant decisions or the principles of selecting students and so on (for an extensive review see OECD, 2007 and 2009).

This great volume of data offers an exciting framework to analyze and identify the potential influence of different variables on results. Although we restrict our analysis to the Spanish case, in 2006, ten Spanish regions decided to take part in the evaluation with an extended representative sample of their population. In Spain, the decision about the quantity of the educational budget and its allocation is the competency of the regions. For this reason, this analysis allows us to evaluate potential efficiency divergences among regions within the same country.

As we mentioned before, the possibility of using information at student level for measuring efficiency has one great advantage over most of the studies completed within the educational context [Waldo (2007)], which usually use aggregate data at country [Alfonso and St. Aubyn (2006)], district [McCarty and Yaisawarng (1993), Banker *et al*. (2004)] or school level [Muñiz (2002), Cordero *et al*. (2008)]. In addition to facilitating the analysis and interpretation of the results from estimations [Summers and Wolfe (1977), Hanushek *et al*. (1996)], it provides information on students' efficiency independently of either the educational system or school efficiency. Furthermore, the measurement of efficiency at student level allows us to separately consider the students' own and their schoolmates' socioeconomic levels, two inputs which cannot be simultaneously included with aggregated data [Santín (2006)].

The paper is organized as follows. Section 1 provides an overview of educational production functions and presents the parametric stochastic distance function and our estimation strategy. In Section 2, the data set and variables selected are described. Section 3 provides results and a discussion of our empirical analysis and the final section offers some conclusions.

1. Education and efficiency measurement with a parametric distance function

1.1. Estimating an educational production function through distance functions

The attempts to estimate educational production functions are based on the analogy between this sector and an industry. In the latter, the firms produce different outputs using inputs such as labour and capital which are transformed according to the existing technology into commodities and/or services. In education, schools produce educational outputs in the form of student achievement and other valued results using facilities, equipment, teachers, students' own characteristics, peer-group interactions, supervisors and administrators. This relationship can be defined with a basic formulation expressed in the following way [Levin (1974), Hanushek (1986)]:

$$A_{is} = f\,(B_{is}, S_{is}, P_{is}, I_{is}) \qquad [1]$$

where A_{is} represents the achievement of student i at school s, usually represented by the results obtained in standardized tests. This output vector depends on a set of factors represented by socioeconomic background (B_{is}), mainly family characteristics, school inputs (S_{is}) such as educational material, teachers or infrastructures in the school, influence of classmates or peer-group effect (P_{is}), and the students' innate abilities (I_{is}).

This function can be estimated statistically using a multivariate regression model. A further refinement of the educational production function would be to construct a frontier production function where only those units that maximize their results according to their resources are placed within the boundary. In this case, instead of using simple econometric analysis to estimate equation [1], more sophisticated methods are required. Following Perelman and Santín (2008), in this paper we use parametric stochastic distance functions at student level in order to analyse production functions in education in greater depth. For this purpose, equation [1] becomes:

$$D_{is} = g\,(A_{is}, B_{is}, S_{is}, P_{is})\, I_{is} \qquad [2]$$

where g represents the best practice technology used in the transformation of educational inputs to outputs, and D_{is} is the distance that separates each student i attending school s from the technological boundary. Unobservable innate student abilities, I_{is}, are assumed to be randomly normally distributed[2] in the population and to influence individual performance in a multiplicative way. This simple transformation places the empirical estimation of equation [2] within the framework of parametric stochastic frontier analysis, which, under specific distributional assumptions, allows the disentangling of educational inputs, random effects and efficiency (distance to the production frontier).

(2) The scoring of modern IQ tests such as the Wechsler Adult Intelligence Scale [Wechsler (2008)], the primary clinical instrument used to measure adult and adolescent intelligence, is now based on a projection of the subject's measured rank on the normal distribution with a central value (average IQ) of 100, and a standard deviation of 15, although not all IQ tests adhere to this standard deviation.

1.2. The parametric stochastic distance function

Defining a vector of inputs $x = (x_1, x_2, ..., x_K) \in \Re^{K+}$ and a vector of outputs $y = (y_1, y_2, ..., y_M) \in \Re^{M+}$, a feasible multi-input multi-output production technology can be defined using the output possibility set $P(x)$, which represents the set of all outputs, $y \in \Re^{K+}$, that can be produced using the input vector, $x \in \Re^{K+}$. That is, $P(x) = \{(x, y)$: *x can produce y*$\}$ and we assume that the technology satisfies the set of microeconomic axioms listed in Fare and Primont (1995) including strong disposability, convexity, closedness and boundedness.

In order to capture efficiency behaviours, the output distance function, introduced by Shephard (1970), can be defined in the output set, $P(x)$, as $D_o(x, y) = \min\{\theta : \theta > 0, (x, y / \theta) \in P(x)\}$. As noted in Fare and Primont (1995), $D_o(x, y)$ is non-decreasing, positively linearly homogeneous and convex in y and non-increasing and quasi-convex in x. The distance function, $D_o(x, y)$, will take a value that is less than or equal to one if the output vector, y, is an element of the feasible production set, $P(x)$. Then, if $D_o(x, y) \leq 1$, the mix (x, y) belongs to the production set $P(x)$ and only when $D_o(x, y) = 1$ is the output vector, y, located on the boundary of the output possibility set[3].

Figure 1 illustrates these concepts in a simple two-output one-input setting. Let us assume that the DMUs A, B, C and D have an equal input endowment to produce outputs y_1 and y_2. Then B and C are efficient because both lie on the boundary of the output possibility set, whereas D and A, as interior points, are inefficient. The measurement of the relative inefficiency of A and D is given by the distance function $\theta_A = OA / OB$ and $\theta_D = OD / OC$.

Our analysis is focused on an output distance function in order to reach our aim of evaluating the behavior of a group of students seeking to obtain the best possible academic results. The definition of the distance function in the educational context is how the achievement vector may be proportionally increased subject to a fixed input vector.

In our study, we assume a translog functional form to estimate the distance function with some properties such as flexibility, ease of calculation and homogeneity of degree +1 behavior[4]. This form has been used previously in other studies such as Lovell *et al*. (1994), Grosskopf *et al*. (1997) or Coelli and Perelman (1999, 2000).

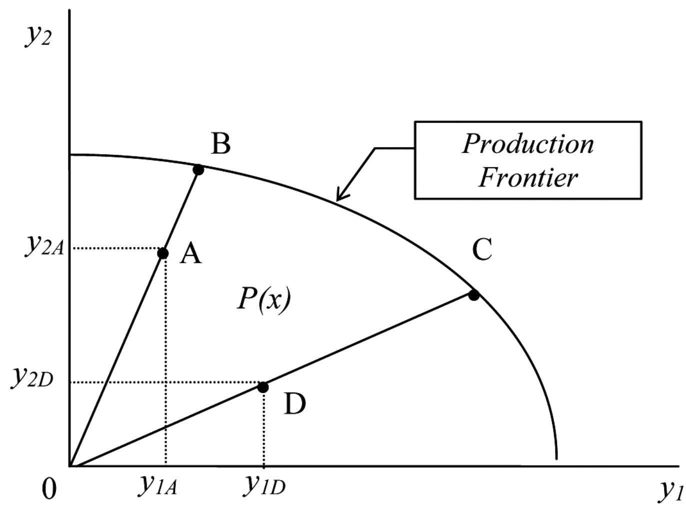
The translog distance function for the case of M outputs and K inputs adopts the following specification:

$$\ln D_{ois}(x,y) = \alpha_0 + \sum_{m=1}^{M} \alpha_m \ln y_{mis} + \frac{1}{2}\sum_{m=1}^{M}\sum_{n=1}^{M} \alpha_{mn} \ln y_{mis} \ln y_{nis} + \sum_{k=1}^{K} \beta_k \ln x_{kis} + \frac{1}{2}\sum_{k=1}^{K}\sum_{l=1}^{K} \beta_{kl} \ln x_{kis} \ln x_{lis} + \sum_{k=1}^{K}\sum_{m=1}^{M} \gamma_{km} \ln x_{kis} \ln y_{mis} \qquad (i = 1, 2, ..., N), (s = 1, 2, ..., H) \quad [3]$$

(3) The distance function may be specified with either input or output orientation. So input distance function analysis could be defined in a similar way imposing an input orientation and given output endowments.

(4) The Cobb Douglas form does not satisfy the concave imposition in the output dimension.

Figure 1: OUTPUT POSSIBILITY SET P(X)



Source: Own elaboration.

where sub-index i denotes the ith pupil in the sample belonging to the sth school, K is the total number of inputs and M the total number of outputs. With the aim of obtaining the frontier surface, we set $D_o(x, y) = 1$, which implies that $D_o(x, y) = 0$. Furthermore, the parameters of the above distance function must satisfy some restrictions of symmetry

$$\alpha_{mn} = \alpha_{nm} \text{ ; m, n = 1, 2, …, M,}$$
$$\beta_{kl} = \beta_{lk} \text{ ; k, l = 1, 2,…, K,}$$

and homogeneity of degree +1 in outputs[5]. The analytical expressions of these restrictions are:

$$\sum_{m=1}^{M} \alpha_m = 1 \;;\quad \sum_{m=1}^{M} \alpha_{mn} = 0 \quad \text{and} \quad \sum_{m=1}^{M} \gamma_{km} = 0 \qquad [4]$$

In order to impose the homogeneity of degree + 1 in outputs, we normalize the output distance function arbitrarily by one of the outputs following Lovell *et al*. (1994), being the expression:

$$\ln D_{ois}(x,y) / \ln y_{Mis} = TL(x_{is}, y_{is} / y_{Mis}, \alpha, \beta, \gamma) \qquad [5]$$

(5) The homogeneity restriction implies that the distance of the unit to the boundary of the production set is measured by radial expansion.

where:

$$TL(x_{is}, y_{is} / y_{Mis}, \alpha, \beta, \gamma) = \alpha_0 + \sum_{m=1}^{M-1} \alpha_m \ln(y_{mis}/y_{Mis}) + \frac{1}{2}\sum_{m=1}^{M-1}\sum_{n=1}^{M-1} \alpha_{mn} \ln(y_{mis} / y_{Mis})$$
$$\ln(y_{nis} / y_{Mis}) + \sum_{k=1}^{K} \beta_k \ln x_{kis} + \frac{1}{2}\sum_{k=1}^{K}\sum_{l=1}^{K} \beta_{kl} \ln x_{kis} \ln x_{lis} + \sum_{k=1}^{K}\sum_{m=1}^{M-1} \gamma_{km} \ln x_{kis} \ln(y_{mis} / y_{Mis}) \quad [6]$$

Rearranging terms, the function above can be rewritten as follows:

$$-\ln(y_{Mis}) = TL(x_{is}, y_{is} / y_{Mis}, \alpha, \beta, \gamma) - \ln D_{ois}(x, y) \quad [7]$$

Following Lovell *et al.* (1994), we can consider the unobservable term –ln $D_{ois}(x, y)$ as a random error term, which is the radial distance from the boundary. Then we can easily obtain the Battese and Coelli (1988) expression of the traditional stochastic frontier model proposed by Aigner, Lovell and Smith (1977) and Meeusen and van den Broeck (1977) considering $u_{is} = -\ln D_{ois}(x, y)$ and adding another term v_{is} which captures noise:

$$-\ln(y_{Mis}) = TL(x_{is}, y_{is} / y_{Mis}, \alpha, \beta, \gamma) + \varepsilon_{is} \quad (\varepsilon_{is} = u_{is} + v_{is}) \quad [8]$$

Notice that the term $u_{is} = -\ln D_{ois}(x, y)$ is a negative random term assumed to be distributed as a semi-normal $|N(0, \sigma_u^2)|$ distribution and the term v_{is} is assumed to be a two-sided random (stochastic) disturbance designed to account for statistical noise and distributed iid v~ $N(0, \sigma_u^2)$. Both terms are independently distributed $\sigma_{uv} = 0$.

In the context of education, three kinds of variables are considered: scores obtained by students in standardized tests (outputs), one vector of educational variables indispensable for achievement (inputs), whose effect on results must be positive, i.e., a greater endowment of any of these variables must have positive impact on results, and finally, a set of variables about which we need to know whether or not they have influence on the educational process since it cannot be known a priori if their effect is positive, negative or inexistent (environmental variables).

Therefore, we opt for the Battese and Coelli (1995) model which proposes a stochastic frontier model in which the inefficiency effects u_{is} are expressed as an explicit function of a vector of environmental variables $z = (z_1, z_2, ..., z) \in \Re^S$ where:

$$u_{is} = \delta_0 + z_{is}\, \delta \quad [9]$$

δ is a vector of parameters that must be simultaneously estimated with the parameters included in equation 8. To the best of our knowledge, this is the first time in the economics of education literature that the Battese and Coelli (1995) model has been employed at student level considering the three sets of educational variables named above. This model enables us to identify the sign of the effect of each environmental variable and its influence on students' levels of efficiency independently of the inputs. We think this framework is appealing in terms of educational policy makers taking decisions in order to get a better distribution and organization of public resources.

1.3. Variance decomposition

Given the purpose of the paper, our main concern is not only to obtain a pure efficiency score for each pupil, net of inputs and environmental variables, but also to identify the causes of the efficiency detected: school efficiency or the students' own efficiency. Most empirical work concentrates on highlighting out only one agent responsible for efficiency, school or student. Nevertheless, in real life, it is doubtful to assume that efficiency is only caused by students (mean efficiency between schools would be exactly the same) or by schools (mean efficiency within schools would be equal and all efficiency variance would be explained by the mean efficiency between schools). In this paper, we follow Perelman and Santín (2008) to decompose student and school efficiency. We are especially interested in disentangling the efficiency attributable to school management of educational resources, since this is a factor in which the public sector can intervene through education policy.

After the estimation of the Battese and Coelli (1995) model depicted above, the decomposition of estimated efficiency may be carried out through an analysis of the variance of the term $\hat{\theta}_{is}$, where $\hat{\theta}_{is} = \hat{u}_{is} - u_{is}$. Following Perelman and Santín (2008), we assume that mean efficiency differences among schools are due to efficiency attributable to schools (between) while differences among students in the same school (within) are due to students' own efficiency[6]. Hence, the decomposition of the efficiency variance can be carried out as follows through a one-way analysis of variance,

$$\hat{S}^2_{u_{is}} = \hat{S}^2_{u_s B} + \hat{S}^2_{u_i W} \qquad [10]$$

Thus, efficiencies between schools ($\hat{S}^2_{u_s B}$) include teachers' characteristics and motivation, the pedagogical methods employed, management strategies or the relationship between parents and principals. On the other hand, efficiencies within school ($\hat{S}^2_{u_i W}$) are attributable to students' dedication and effort. We expect efficiency to be a mix of the two components.

1.4. Elasticity estimations

One advantage of the parametric distance function is that this technique allows us to calculate the output and input elasticities which give us relevant information about the effect of each input on each output. A peculiarity of translog distance functions is that the elasticity value is different in each observed unit, so it is necessary to obtain the elasticity for each point. As is usual in educational stud-

(6) If the input and control variables depicted in Equations 8 and 9 control for the other determinants of achievement (mainly the student's background, school variables, peer group effect and other characteristics or environmental variables), then the remaining efficiency effect depends only on the student and the school. Hence, we implicitly assume that (after controlling for x_{is} and z_{is}) the student's outcomes and efficiency are independent variables. However, it is worth noting that a possible selection bias could arise if students are not distributed over schools independently of their potential efficiencies. This could happen if the most efficient students are concentrated in the most efficient schools or if the most efficient schools could select the most efficient students. What we assume in this paper is that the x_{is} and z_{is} variables influence y_{is} but are independently distributed for u_{is}.

ies, we analyse the distance function elasticity with respect to inputs and outputs and the rate of change inputs and outputs. For these purposes, we use the following expressions:

$$r_{D,x_k} = \frac{\partial D}{\partial x_k} = \frac{\partial \ln D(x,y)}{\partial \ln x_k}\frac{D(x,y)}{x_k} \quad ; \quad r_{D,y_m} = \frac{\partial D}{\partial y_m} = \frac{\partial \ln D(x,y)}{\partial \ln y_m}\frac{D(x,y)}{y_m} \qquad [11]$$

where positive values of r_{D,x_k} (r_{D,y_m}) indicate that an increase in the input (output) implies a higher inefficiency (efficiency).

Expressions of partial elasticities between output “m” and input “k”, which indicate the variation in output “m” level with an increase in the input “k” proportion, and the variation of one output “n” with respect to another “m”, which can be interpreted as the extent that output “n” changes with an increase in output “m”, are as follows:

$$s_{y_m,x_k} \equiv \frac{dy_m/y_m}{dx_k/x_k} = \frac{r_{D,x_k}}{r_{D,y_m}} = \frac{\beta_k + \sum_{k=1}^{K}\beta_{kl}\ln x_k + \sum_{m=1}^{M}\delta_{km}\ln y_m}{\alpha_m + \sum_{m=1}^{M}\alpha_{mn}\ln y_m + \sum_{k=1}^{K}\delta_{km}\ln x_l} \qquad [12]$$

$$s_{y_m,y_n} \equiv \frac{dy_n/y_n}{dy_m/y_m} = -\frac{r_{D,y_m}}{r_{D,y_n}} = \frac{\alpha_m + \sum_{m=1}^{M}\alpha_{mn}\ln y_m + \sum_{k=1}^{K}\delta_{km}\ln x_k}{\alpha_n + \sum_{n=1}^{M}\alpha_{mn}\ln y_n + \sum_{k=1}^{K}\delta_{kn}\ln x_k} \qquad [13]$$

A positive sign in equation [12] means that an increase in input “k” produces an increase in output “m”. The interpretation is the opposite for the case of a negative sign. In equation [13], a negative sign entails that an increase in output “m” produces a decrease in output “n”, and the opposite in the case of a positive sign.

2. Analisys of spanish results in pisa 2006

2.1. Data

In our empirical analysis, we use Spanish data from PISA 2006 which provides us with data from 15 year-old students belonging to ten regions that decided to take part in the evaluation with an extended representative sample of their population[7] (Andalusia, Aragon, Asturias, Cantabria, Castile-Leon, Catalonia, Galicia, La Rioja, Navarre, Basque Country) and a group labelled as ‘other regions’ made up of the seven remaining Spanish regions. It is worth noting here, that the

(7) In 2003, three regions took part in the evaluation (Castile-Leon, Catalonia and the Basque Country). Perelman and Santín (2008) also analyse Spanish data from PISA 2003 but they do not study regional differences in efficiency, which is very informative for the case of Spain since education funding is totally decentralized.

Spanish Autonomous Communities (hereafter the regions) have been fully responsible for the management of educational resources in Spain since 2000. Therefore, they should be the ones most interested in analysing PISA results as a previous step to the application of more effective educational policies. To perform this analysis, we have data about 19,605 students and 685 schools distributed across eleven regions as shown in Table 1. Schools can be divided into three groups according to the type of ownership: public (financed by the government), private (government independent) and government dependent (private management and financed by the government).

Table 1: Distribution of students and schools by ownership and region

Region	Students	Schools	Public	Semi-Private	Private
Andalusia	1,463	51	37	13	1
Aragon	1,526	51	31	16	4
Asturias	1,579	51	31	14	8
Cantabria	1,496	53	31	19	3
Castile-Leon	1,512	52	31	17	4
Catalonia	1,527	51	29	11	10
Galicia	1,573	51	36	11	6
La Rioja	1,333	51	22	20	3
Navarre	1,590	51	30	19	3
Basque Country	3,929	150	63	83	4
Other regions	2,077	74	44	20	10
Spain	19,605	685	385	243	57

Source: PISA 2006 Report for Spain.

One of the main advantages of the PISA study is that it does not evaluate cognitive abilities or skills through one single score but each student receives a score in each test within a continuous scale. In this way, PISA attempts to collect the effect of particular external conditioning factors affecting the students during the test. Furthermore, it also means that measurement error in education is not independent of the position of the student in the distribution of results. Precisely, students with very low or high results have higher associated measurement errors and higher asymmetry in error distribution.

Likewise, given that school factors, home and socioeconomic context play an important role in students' learning, PISA also collects an extensive dataset on these variables through two questionnaires: one completed by the students themselves and another one filled out by school principals. From these data, it is possible to extract a great amount of information referring to the main determining driver factors of educational performance represented by variables associated with family and educational environments as well as with school management and educational supply.

2.2. Variables

To perform the efficiency analysis, we use three sets of variables: outputs, inputs and environmental factors. As output indicators we have used test scores as is usual in most studies in education. However, the selection of inputs and exogenous variables can be complex and, in some cases, even confusing. Given that the literature does not provide an explicit rule to discriminate between them, in this study we have based our decision on the following criteria. First, input variables must fulfil the requirement of isotonicity (i.e., *ceteris paribus*, more input implies an equal or higher level of output). Thus, the selected input variable should present a significant positive correlation with the output vector in addition to having theoretical support in previous works. Second, input variables should be objective measures of educational resources or subjective opinions that could be checked by an external auditor. Third and finally, categorical and binary variables that divide the sample into different subgroups are considered as environmental factors to explain efficiency ex-post.

Outputs and plausible values

The true output as the result of an individual education is very difficult to measure empirically due to its inherent intangibility. Education does not consist only of the ability to repeat information and answer questions, but also involves the skills to interpret information and learn how to behave in society. Unfortunately, it is really difficult to measure all of these factors. In spite of the multi-product nature of education, most studies have used the results obtained in cognitive tests since they are difficult to manipulate and respond to administration demands. But perhaps, as Hoxby (2000) states, the most important reason could be that both policy makers and parents use this criterion to evaluate educational output to choose the school for their children and even their place of residence.

In this study we use the results obtained by students in the three competences evaluated in PISA (mathematics, reading comprehension and sciences) as the vector of educational output. As has already been mentioned, PISA uses the concept of plausible values to measure the performance of students, since measures in these subjects have a wide margin of error due to the fact that the measuring concept is abstract and is subject to the special circumstances of students and their environment on the date of their exams. Moreover, questions about educational knowledge may have different levels of difficulties and the measuring error is dependent on the student's position in the distribution of performance results. Students with a very high result suffer higher measuring error and higher asymmetry in their distribution than students with an average result. For this reason PISA 2006 used measures based on the Rasch model [Rasch (1960), Wright and Masters (1982)], which uses plausible values instead of working with a particular mean value for each student's knowledge. These values are random values obtained from the distribution function of results estimated from the answers in each test. They can be interpreted as a representation of the ability range of each student[8] [Wu and Adams (2002)].

(8) For a review of plausible values literature, see Mislevy *et al*. (1992). For a concrete Studio of Rasch model and how to obtain feasible values in PISA, see OECD (2005).

Table 2 reports the average value and standard deviation for plausible values of the three tests (math, reading comprehension and sciences) in each region. Plausible values in the three tests are used as outputs in the efficiency analysis. In order to obtain correct results and avoid problems of bias in the estimations, it will be necessary to calculate five different efficiency measures for each trio of plausible values and take the mean value afterwards, instead of using mean values to obtain one efficiency measure [OECD (2005)].

It is worth noting here that the standard deviation in the results offers additional information about the equity of the educational system. For example, although La Rioja and Castile-Leon are the top performers in Spain, the results in Castile-Leon, where the standard deviation is considerable lower than in La Rioja, seem preferable from a public policy point of view. According to this reasoning we can conclude that the distribution of the test scores is quite similar across all the Spanish regions. Thus, it can be assumed that, in Spain, there is no clear trade-off between high scores and equity.

Inputs

In order to carry out the distance function efficiency analysis, we have used three different inputs that are directly involved with student learning (ESCS, SCMATEDU and PEER) together with a set of control variables. Table 3 presents a brief description of each variable and Table 4 reports the main descriptive statistics of inputs and environmental variables by regions.

B_{is} in Equation 2 represents the index of the school's educational resources (SCMATEDU). This variable was computed on the basis of seven items measuring the school principal's perceptions of potential factors hindering instruction at school (science laboratory equipment, instructional materials, computers for instruction, internet connectivity, computer software for instruction, library materials and audio-visual resources). The items were inverted for scaling so more positive values of this index indicate higher levels of educational resources[9].

ESCS reflects the socio-economic background of each student. It is an index of the economic, social and cultural status of students created by PISA analysts from three variables related to family background from students' questionnaire: the index of highest level of parental education in number of years of education according to the International Standard Classification of Education [ISCED, OECD (1999)], the index of highest parental occupation status according to International Socio-economic index of Occupational Status [ISEI, Ganzeboom *et al.* (1992)] and the index of educational possessions at home. Finally, PEER incorporates information about the characteristics of students' classmates[10]. This variable is defined by the average of the ESCS variable of students that belong to the same school of the evaluated individual.

(9) This variable shows a significant and positive correlation with the three outputs.

(10) For a review of the effect of these variables on the results, see Betts and Shkolnik (2000) or Hanushek *et al.* (2001).

Table 2: Plausible Values in Sciences, Math and Reading

		Plausible Values Science				
		Sci_1	Sci_2	Sci_3	Sci_4	Sci_5
Andalusia	mean	481.63	482.36	481.29	481.07	481.33
	st-deviation	(87.13)	(86.66)	(87.39)	(89.70)	(87.83)
Aragon	mean	514.86	515.52	516.37	515.99	516.86
	st-deviation	(87.60)	(87.29)	(87.79)	(87.50)	(87.64)
Asturias	mean	513.26	511.89	513.40	513.12	513.55
	st-deviation	(81.08)	(82.46)	(82.55)	(82.14)	(83.66)
Cantabria	mean	514.46	514.17	513.43	514.16	513.86
	st-deviation	(84.05)	(83.58)	(84.26)	(83.56)	(85.35)
Castile-Leon	mean	524.47	523.33	522.24	522.53	523.11
	st-deviation	(78.12)	(79.06)	(79.30)	(80.00)	(79.42)
Catalonia	mean	493.71	494.60	496.02	494.47	495.33
	st-deviation	(89.46)	(88.23)	(90.52)	(87.97)	(88.01)
Galicia	mean	506.45	507.37	507.29	507.17	507.26
	st-deviation	(87.33)	(85.90)	(85.66)	(86.59)	(86.65))
La Rioja	mean	522.54	520.89	522.62	521.47	522.02
	st-deviation	(87.25)	(88.63)	(85.40)	(87.36)	(88.06)
Navarre	mean	511.95	511.87	512.06	512.04	512.51
	st-deviation	(87.41)	(87.06)	(88.33)	(88.72)	(89.62)
Basque Country	mean	497.62	498.22	497.38	497.34	498.32
	st-deviation	(81.80)	(83.17)	(83.18)	(83.64)	(82.97)
Other Regions	mean	490.69	491.18	490.69	489.26	491.70
	st-deviation	(90.40)	(90.66)	(91.00)	(91.34)	(90.19)
Total Spain	mean	504.92	505.00	504.99	504.63	505.38
	st-deviation	86.24	(86.37)	(86.68)	(86.99)	(86.90)

Source: PISA 2006 Report for Spain.

Table 2: PLAUSIBLE VALUES IN SCIENCES, MATH AND READING (continuation)

		Plausible Values Math				
		Math_1	Math_2	Math_3	Math_4	Math_5
Andalusia	mean	470.11	470.01	470.31	469.46	470.51
	st-deviation	(83.41)	(84.11)	(84.88)	(85.34)	(84.77)
Aragon	mean	514.59	514.86	516.71	515.62	515.87
	st-deviation	(97.05)	(96.56)	(96.03)	(96.04)	(96.28))
Asturias	mean	5501.45	500.45	502.31	501.15	502.60
	st-deviation	(80.75)	(81.10)	(80.81)	(79.82)	(81.83)
Cantabria	mean	506.60	506.23	505.87	506.12	506.05
	st-deviation	(84.34)	(83.79)	(85.22)	(83.65)	(85.23)
Castile-Leon	mean	519.91	519.41	518.99	517.80	518.65
	st-deviation	(81.21)	(81.92)	(81.36)	(82.15)	(81.59)
Catalonia	mean	488.91	489.77	491.50	489.65	490.16
	st-deviation	(85.72)	(85.80)	(87.13)	(85.82)	(85.10)
Galicia	mean	496.52	496.70	496.50	496.75	496.13
	st-deviation	(82.53)	(82.19)	(82.14)	(82.75)	(82.02)
La Rioja	mean	526.59	526.31	526.69	525.66	526.83
	st-deviation	(87.02)	(88.78)	(84.97)	(87.73)	(87.13)
Navarre	mean	517.06	519.43	518.88	519.02	519.09
	st-deviation	(88.38)	(88.69)	(88.80)	(90.05)	(90.96)
Basque Country	mean	504.31	504.97	503.85	503.27	504.80
	st-deviation	(83.06)	(83.72)	(84.23)	(84.80)	(84.07)
Other Regions	mean	479.38	480.15	480.18	478.83	481.32
	st-deviation	(87.01)	(86.86)	(87.22)	(87.71)	(87.70)
Total Spain	mean	501.80	502.13	502.27	501.51	502.43
	st-deviation	¡(86.74)	(87.02)	(87.04)	(87.37)	(87.30)

Source: PISA 2006 Report for Spain.

Table 2: Plausible Values in Sciences, Math and Reading (continuation)

		Plausible Values Reading				
		Read_1	Read_2	Read_3	Read_4	Read_5
Andalusia	mean	452.41	453.72	451.72	451.88	452.13
	st-deviation	(85.86)	(84.33)	(85.02)	(86.19)	(85.48)
Aragon	mean	484.89	485.11	485.78	485.11	485.00
	st-deviation	(86.90)	(86.91)	(86.12)	(86.23)	(86.31)
Asturias	mean	482.51	481.39	483.00	482.17	481.56
	st-deviation	(80.52)	(81.75)	(81.92)	(83.02)	(82.56)
Cantabria	mean	479.06	479.27	477.99	479.38	478.83
	st-deviation	(84.83)	(83.78)	(84.46)	(83.99)	(85.01)
Castile-Leon	mean	481.39	480.72	480.14	479.71	481.07
	st-deviation	(74.26)	(75.13)	(74.80)	(76.38)	(75.04)
Catalonia	mean	478.56	480.16	480.46	480.36	479.40
	st-deviation	(88.98)	(88.30)	(90.31)	(87.95)	(88.09)
Galicia	mean	482.18	482.38	482.27	482.05	482.08
	st-deviation	(88.84)	(88.82)	(88.39)	(88.29)	(87.76)
La Rioja	mean	496.11	494.37	495.51	494.82	494.55
	st-deviation	(82.14)	(80.48)	(80.76)	(81.79)	(82.63)
Navarre	mean	482.04	481.80	481.72	480.99	481.80
	st-deviation	(79.03)	(78.48)	(77.89)	(80.59)	(79.38)
Basque Country	mean	491.18	491.01	490.12	490.65	491.75
	st-deviation	(86.08)	(87.65)	(87.34)	(86.98)	(87.25)
Other Regions	mean	462.30	462.40	462.21	461.30	461.85
	st-deviation	(85.30)	(86.52)	(85.26)	(86.05)	(85.44)
Total Spain	mean	480.24	480.21	479.98	479.83	480.10
	st-deviation	(85.07)	(85.21)	(85.20)	(85.55)	(85.38)

Source: PISA 2006 Report for Spain.

Table 3: VARIABLE DEFINITIONS

Variable	Description
Inputs	
SCMATEDU	Index of the quality of the school's educational resources
ESCS	Index of economic, social and cultural status
PEER	Average ESCS index of the student's peer group
Z's	
PRIVATE	Attending a private school (1 = yes; 0 = no)
GOVDEP	Attending a government-dependent school (1 = yes; 0 = no)
ZCHLSIZE	Number of students in school
STRATIO	Weighted number of teachers divided by total number of students
REPEAT ONCE	The student has repeated a shool year once (1 = yes; 0 = no)
REPEAT MORE	The student has repeated a shool year more than once (1 = yes; 0 = no)
INMIGRANT 1	The student and at least one of the parents were born abroad
INMIGRANT 2	The student was born in Spain but at least one of the parents was not
REGIONS	Belong to a region (ten different dummy variables)

Source: PISA 2006 Report.

In addition to input variables, we have considered that other factors related to the characteristics of schools and students may influence efficiency in education (z´s variables). In particular, we have analyzed the effect of the following:

- School ownership. This variable has been included in the analysis in order to test whether the public, government-dependent private or private schools have some influence over students' efficiency. Regarding this issue, in the literature we can find evidence that supports the idea of better performance in private schools [Chubb and Moe (1990), Sander (1996), Figlio and Stone (1997), Neal (1997), McEwan (2001)] while others do not find enough evidence to justify this superiority [Witte (1992), Goldhaber (1996), Vandenberghe and Robin (2004), Mancebón and Muñiz (2007)]. In our case, we have included this information using public schools as the reference. Two dummy variables have been defined: PRIVATE, which takes the value one if the school is private and zero otherwise, and GOVDEP, which takes the value one if the school is government-dependent and zero otherwise.
- School size (SCHLSIZE): This variable indicates the total number of students in the school. The influence of this variable in the educational process has also been tested in previous studies where we can find some results supporting that schools with more students have better results [Bradley and Taylor (1998), Barnett *et al*. (2002)] but others that conclude that this factor does not affect the results [Hanushek and Luque (2003)].

- Classroom size (STRATIO): This variable is the ratio between the total number of students in the school (SCHLSIZE) and the total number of teachers weighted according to their dedication (part-time teachers contribute 0.5 and full-time teachers 1). This variable is usually considered a school input in efficiency analysis because of the results of some studies in which a direct relationship is found between reduced groups and higher academic performance [Card and Krueger (1992), Hoxby (2000), Krueger (2003)]. However, other studies conclude that this variable is not significant [Hanushek (1997 and 2003), Pritchett and Filmer (1999)]. Given that the linear correlation between this variable and the output is, contrary to expectations, positive, we decided, in order to avoid potential bias in the estimation, to consider this information as an environmental variable in the efficiency analysis, instead of considering it as an input.
- Immigrant status. This factor, whose influence has received increasing attention in the literature in recent years [Gang and Zimmermann (2000), Entorf and Minoiu (2005), Cortes (2006)], is especially interesting for Spain as a consequence of the huge growth of the immigrant population of school age during the last decade[11]. In view of this phenomenon, several studies have recently studied the influence of this factor on the results of Spanish students by using information provided by the PISA database [Calero and Escardibul (2007), Zinovyeva *et al*. (2008)]. In our study, this factor has been included through two dummy variables (INMIGRANT1 and INMIGRANT2) that allow us to identify the first and second order (the student and his/her parents were born abroad; the student was born in Spain but at least one of the parents was born abroad) immigrant status.
- Repeat Once and Repeat More are two dummy variables that represent students that have repeated one or more than one school year, respectively. There is a vast literature on the effect of repetition on academic performance and self-esteem with the majority of educational researchers concluding that it is negative [Holmes (1989), Jimerson *et al*. (2002)]. This phenomenon may be of great significance in the case of Spain, where the repetition rate is much higher than in other countries in the OECD[12]. Obviously, it is expected that to be a repeater will result in a lower efficiency index although our aim is to quantify this effect after controlling for the different inputs and environmental factors considered.
- Regions. In order to test whether there are significant differences across regions in terms of inefficiency, ten different dummy variables have been constructed (one for each region with a representative sample), taking the value one if the student belongs to a particular region and zero otherwise. Each region is compared with the sample of students belonging to the other regions.

(11) According to official Spanish statistics captured by the MEC (2008), foreign students in non-university education have grown from a total number of 72,335 in 1998 to 695,190 in 2008.

(12) In Spain, 40% of the students have repeated a school year at least once [Fuentes (2009)].

Table 4: DESCRIPTIVE STATISTICS OF INPUTS AND ENVIRONMENTAL VARIABLES

Region	Observ.	Statistic	ESCS	SCMATEDU	PEER	Private (%)	Semi-Priv (%)
Andalusia	1,463	Mean	5.508	4.050	5.488	0.023	0.243
		St. Dev.	(1.075)	(1.012)	(0.548)	(0.148)	(0.429)
Aragon	1,526	Mean	5.957	4.632	6.024	0.088	0.299
		St. Dev.	(1.016)	(0.892)	(0.479)	(0.282)	(0.458)
Asturias	1,579	Mean	5.967	4.605	6.010	0.156	0.238
		St. Dev.	(1.023)	(0.920)	(0.545)	(0.363)	(0.426)
Cantabria	1,496	Mean	5.933	4.438	5.965	0.063	0.331
		St. Dev.	(0.970)	(0.821)	(0.452)	(0.243)	(0.471)
Castile-Leon	1,512	Mean	5.889	4.657	5.863	0.089	0.304
		St. Dev.	(1.014)	(0.945)	(0.472)	(0.285)	(0.460)
Catalonia	1,527	Mean	5.913	4.675	5.944	0.232	0.220
		St. Dev.	(1.049)	(1.024)	(0.585)	(0.422)	(0.414)
Galicia	1,573	Mean	5.745	4.218	5.766	0.114	0.190
		St. Dev.	(1.048)	(0.890)	(0.596)	(0.318)	(0.393)
La Rioja	1,333	Mean	5.972	4.665	5.992	0.061	0.424
		St. Dev.	(0.989)	(0.855)	(0.449)	(0.239)	(0.494)
Navarre	1,590	Mean	5.947	4.690	5.884	0.054	0.383
		St. Dev.	(1.007)	(0.910)	(0.519)	(0.225)	(0.486)
Basque Country	3,929	Mean	6.062	4.517	6.107	0.019	0.581
		St. Dev.	(0.981)	(0.896)	(0.512)	(0.137)	(0.493)
Other Regions	2,077	Mean	5.894	4.443	5.920	0.141	0.296
		St. Dev.	(1.084)	(0.985)	(0.642)	(0.348)	(0.457)
SPAIN	19,605	Mean	5.494	3.209	5.760	0.087	0.350
		St. Dev.	(0.885)	(0.119)	(0.368)	(0.282)	(0.477)

Source: Personal compilation based on PISA 2006 data for Spain.

Table 4: Descriptive statistics of inputs and environmental variables (continuation)

Region	Observ.	Statistic	School Size	Teacher-Student Ratio	Repeat once	Repeat more	Inmig1	Inmig2
Andalusia	1,463	Mean	700.88	13.51	0.322	0.091	0.027	0.073
		St. Dev.	(356.82)	(4.059)	(0.467)	(0.288)	(0.163)	(0.256)
Aragon	1,526	Mean	708.24	12.12	0.279	0.064	0.068	0.096
		St. Dev.	(412.82)	(3.953)	(0.448)	(0.245)	(0.253)	(0.294)
Asturias	1,579	Mean	645.18	11.44	0.252	0.052	0.034	0.093
		St. Dev.	(336.62)	(4.603)	(0.434)	(0.222)	(0.182)	(0.291)
Cantabria	1,496	Mean	619.23	11.46	0.298	0.058	0.055	0.102
		St. Dev.	(257.44)	(4.640)	(0.457)	(0.234)	(0.227)	(0.303)
Castile-Leon	1,512	Mean	717.15	12.07	0.282	0.056	0.038	0.067
		St. Dev.	(390.38)	(3.938)	(0.450)	(0.229)	(0.192)	(0.250)
Catalonia	1,527	Mean	636.09	12.35	0.242	0.028	0.099	0.153
		St. Dev.	(283.75)	(3.408)	(0.428)	(0.166)	(0.299)	(0.359)
Galicia	1,573	Mean	517.31	10.49	0.277	0.100	0.051	0.110
		St. Dev.	(261.76)	(3.982)	(0.447)	(0.300)	(0.220)	(0.314)
La Rioja	1,333	Mean	611.76	13.10	0.270	0.048	0.070	0.101
		St. Dev.	(363.27)	(4.461)	(0.444)	(0.214)	(0.255)	(0.301)
Navarre	1,590	Mean	700.04	10.78	0.216	0.038	0.081	0.122
		St. Dev.	(424.33)	(3.577)	(0.412)	(0.192)	(0.273)	(0.328)
Basque Country	3,929	Mean	784.88	11.99	0.184	0.035	0.048	0.077
		St. Dev.	(518.17)	(4.733)	(0.388)	(0.183)	(0.214)	(0.267)
Other Regions	2,077	Mean	764.63	13.36	0.291	0.058	0.094	0.159
		St. Dev.	(344.37)	(5.263)	(0.454)	(0.233)	(0.292)	(0.366)
SPAIN	19,605	Mean	689.49	12.07	0.255	0.055	0.060	0.103
		St. Dev.	(395.23)	(4.45)	(0.436)	(0.227)	(0.238)	(0.304)

Source: Personal compilation based on PISA 2006 data for Spain.

3. Results

In this section, we present the main results obtained in our analysis. We estimate five output distance functions, one for each trio of plausible values, assuming a stochastic translog technology to measure students' efficiency in PISA 2006. The first step is to impose a homogeneity condition by selecting students' performance in math (y_1) as the dependent variable and the ratios (y_2 / y_1) and (y_3 / y_1) as explanatory variables instead of y_2 and y_3 (students' performance in reading and sciences, respectively)[13].

In order to facilitate the interpretation of the parameters, the original variables were transformed into deviations from the mean values, so first order parameters should be interpreted as the partial elasticity at mean values. Table 5 shows the results after averaging the five estimations.

Therefore, mathematics, reading and science parameters are all positive, which means that efficiency increases when, ceteris paribus, the performance in these subjects improves. The opposite happens with input coefficients, which are all negative and significant, indicating that an input expansion leads to a reduction in student efficiency keeping the output vector fixed. For this estimation, we consider the model without separability between inputs and outputs because most of the input-output cross-products coefficients are statistically significant. The average efficiency, computed as $E[\exp(-u_i \mid \varepsilon)]$, is 0.82, indicating average student efficiency in Spain. The inputs and environmental variables in the model explain about half of the total variance[14].

The results derived from the analysis of z's variables allow us to draw some interesting conclusions. The first relevant idea is that class size has no effect on inefficiency. In fact, we find weak, but significant at 90%, evidence that more students per teacher provides better efficiency[15]. This result has strong implications for the educational policies implemented by many Spanish regional governments for reducing class size in schools.

The second piece of evidence is that variables related to course repetition show a clear negative relation with efficiency scores, even higher when the student has repeated more than one academic year[16]. This result is important from the viewpoint of educational policy since it raises questions about the convenience of repetition policies and their conditioning factors. There are multiple school, family and individual characteristics associated with an increased likelihood of repeating. For instance, simply repeating a grade is unlikely to address the combination of factors that contribute to low achievement or socio-emotional

(13) Following Lovell *et al*. (1994) homogeneity of degree +1 may be imposed if one arbitrary output is chosen and setting $w = 1 / y_M$, one obtains $D_o(x, y / y_M) = D_o(x, y) / y_M$.

(14) To compute the goodness of fit in the model we use Coelli and Perelman (2001).

(15) Calero and Escardibul (2007) also obtain this non expected result between class size and PISA tests scores.

(16) Eide and Showalter (2001) and Corman (2003) obtained similar results using data from the United States.

Table 5: AVERAGE OF THE FIVE PARAMETRIC OUTPUT DISTANCE FUNCTION ESTIMATIONS

Variables	Coeff	St. Dev	t-ratio	Variables	Coeff	St. Dev	t-ratio
Intercept	-0.1969	0.004	-45.91	$(Lnx_2)(Lny_2)$	-0.0330	0.055	-0.61
Lny_1 (mathematics score)	0.4219*			$(Lnx_2)(Lny_3)$	0.1710	0.075	2.30
Lny_2 (reading score)	0.3014	0.009	32.91	$(Lnx_3)(Lny_1)$	0.1159*		
Luy_3 (science score)	0.2767	0.012	22.58	$(Lnx_3)(Lny_2)$	0.6005	0.110	5.48
$(Lny_1)^2$	1.9146*			$(Lnx_3)(Lny_3)$	-0.7164	0.142	-5.06
$(Lny_2)^2$	0.0995	0.008	11.73	z's variables*			
$(Lny_3)^2$	1.1993	0.046	25.95	Intercept	0.2269	0.030	7.52
$(Lny_1)(Lny_2)$	-0.4074*			Repeat once	0.2317	0.007	31.75
$(Lny_1)(Lny_3)$	-1.5072*			Repeat more	0.3738	0.010	38.73
$(Lny_2)(Lny_3)$	0.3079	0.028	9.10	Gov-Dep	0.0123	0.009	1.40
Inputs				Private	-0.0045	0.012	-0.37
Lnx_1 (Scmatedu)	-0.0100	0.004	-2.23	LN School size	-0.0141	0.005	-2.99
Lnx_2 (ESCS)	-0.1265	0.007	-19.39	Inmig1	0.0511	0.011	4.74
Lnx_3 (EFCO)	-0.1169	0.014	-8.25	lnmig2	0.0086	0.009	0.94
$(Lnx_1)^2$	0.0041	0.002	2.29	LN Stratio	-0.0221	0.013	-1.75
$(Lnx_2)^2$	0.1008	0.050	2.01	Andalusia	-0.0136	0.010	-1.31
$(Lnx_3)^2$	-0.2709	0.205	-1.31	Aragon	-0.0855	0.011	-8.08
$(Lnx_1)(Lnx_2)$	-0.0072	0.012	-0.59	Asturias	-0.0559	0.010	-5.33
$(Lnx_1)(Lnx_3)$	0.0013	0.026	0.05	Cantabria	-0.0741	0.011	-6.93
$(Lnx_2)(Lnx_3)$	0.0582	0.077	0.76	Castile-Leon	-0.1017	0.011	-9.40
Input-output				Catalonia	-0.0052	0.010	-0.51
$(Lnx_1)(Lny_1)$	-0.0082*			Galicia	-0.0901	0.011	-8.47
$(Lnx_1)(Lny_2)$	-0.0229	0.016	-1.40	La Rioja	-0.1164	0.012	-9.66
$(Lnx_1)(Lny_3)$	0.0311	0.024	1.29	Navarre	-0.0663	0.011	-6.03
$(Lnx_2)(Lny_1)$	-0.1380*			Basque Country	-0.0185	0.009	-2.13
Sigma-squared	0.0256	0.001	39.48	Expected mean efficiency			0.82
Gamma	0.7796	0.011	71.66	R^2			0.51

Source: Personal compilation based on PISA 2006 data for Spain.

Note: Parameters with (*) are calculated by applying imposed homogeneity conditions.

adjustment problems. Therefore, it seems to be more reasonable to focus on early intervention strategies, especially for students at risk of poor performance.

Thirdly, as we expected, the immigrant condition has a negative influence on efficiency scores, although this relationship is only significant for first generation immigrants, being non-significant for second-generation immigrants[17]. These results reveal the need to implement specific policies aimed at improving the academic performance of these students.

Fourthly, school ownership is not significant so it does not contribute to explaining the student efficiency. In other words, once school, student and environmental variables are taken into account, we cannot conclude that ownership matters for explaining differences in efficiency. And finally, the students from all regions (with the exception of Catalonia and Andalusia) perform better in terms of efficiency than the students belonging to the sample of the other Spanish regions. From our point of view, there is no clear pattern to explain these results. Since 2000, the educational system in Spain has been totally decentralized to the regional governments that decide, independently of the central government, the amount of resources devoted to education. Efficiency analysis allows us to identify best performers in order to learn and apply their successful educational policies in other regions. It seems that La Rioja, Castile-Leon, Galicia and Aragon are the benchmark regions.

Once the initial efficiency analysis and the second stage analysis have been carried out, we may go on to calculate the percentage of student inefficiency directly attributable to their schools once the effect of the exogenous variables has been discounted. For this purpose and following Equation 10, we have completed an analysis of variance of the results obtained at student level that allows us to identify differences in the average efficiency of students belonging to different schools (between-school variance), which can be attributed to school managerial inefficiency, and the variance among students belonging to the same school (within-school variance).

Results reported in Table 6 show that the most important proportion of inefficiency detected depends on the student. Average school inefficiency is almost 13 percent, which denotes that school quality is quite uniform in Spain. Coinciding with the comments made in Section 2.2, it again seems that Spain has a strong equality of educational opportunities in terms of school choice. This means that when parents are choosing a school for their children they should not expect high efficiency differences among the schools considered. However, some significant divergences among regions can be detected. Whereas Andalusia, Galicia and Cantabria present a figure of around 8.5 percent, the Basque Country has a school variance of 25 percent. The causes for this relatively high value for the Basque Country can be found in higher levels of school choice and a current process of yardstick competition since the proportion of government-dependent schools in that region is the highest in the country.

(17) This result may be conditioned by the low number of observations that have the value of one in this variable, since in Spain there are few second order immigrants yet.

Table 6: VARIANCE ANALYSIS

Region	Between (school)	Within (student)	Nº Observations Schools	Students	F-test*
Andalusia	8.66	91.34	51	1,463	2.638
Aragon	11.48	88.52	51	1,526	3.806
Asturias	12.01	87.99	53	1,579	3.991
Cantabria	8.53	91.47	53	1,496	2.565
Castile-Leon	10.24	89.76	52	1,512	3.259
Catalonia	16.16	83.84	51	1,527	5.648
Galicia	8.57	91.43	53	1,537	2.728
La Rioja	13.34	86.66	45	1,333	4.502
Navarre	11.04	88.96	52	1,590	3.733
Basque Country	25.10	74.90	150	3,929	8.357
Other Regions	17.00	83.00	74	2,077	5.588
Mean	12.92	87.08	685	19,605	

Source: Personal compilation based on PISA 2006 data for Spain.

*All F-test present statistical signification at 99%.

Finally, with regard to elasticity estimations, we only report inter-quartile values for the sake of simplicity because we have an elasticity value for each student as was discussed in Section 1.4. Table 7 reports the input-output elasticities. It can be noticed that all the variables have a positive influence on scores, although it is slight in the case of SCMATEDU. Furthermore, the variations in outputs over inputs are different depending on the discipline. On the one hand, the median elasticity of the ESCS in reading is 0.42, 0.28 in Math and 0.41 in Sciences. The average elasticity of PEER in mathematics, reading and sciences is 0.2689, 0.3784 and 0.3823, respectively. Again, an educational policy to avoid the concentration of students with a low socioeconomic background may be more productive than investing more in educational resources.

4. CONCLUSIONS

In this paper we have analyzed the differences in Spanish students' results in PISA 2006 through an educational frontier framework. We have implemented an efficiency analysis using data at student level and considering information about Spanish regions that participate in this study and school ownership in these regions. To the best of our knowledge, this is the first paper that analyzes the results of Spanish students in PISA 2006 using individual data and the Battese and Coelli (1995) model.

Table 7: OUTPUT/INPUT DERIVATES[18]

	Math Inter-quartiles		
	25%	50%	75%
Output with respect to inputs			
SCMATEDU	0.0153	0.0213	0.0303
ESCS	0.2338	0.2845	0.3976
PEER	0.1403	0.2689	0.4400
	Reading Inter-quartiles		
	25%	50%	75%
Output with respect to inputs			
SCMATEDU	0.0229	0.0299	0.0397
ESCS	0.3221	0.4216	0.5581
PEER	0.2228	0.3784	0.5584
	Science Inter-quartiles		
	25%	50%	75%
Output with respect to inputs			
SCMATEDU	0.0209	0.0313	0.0508
ESCS	0.2636	0.4101	0.6845
PEER	0.1897	0.3823	0.6811

Source: Personal compilation based on PISA 2006 data for Spain.

Because of the uncertain environment of the educational production function, we apply a stochastic parametric distance function methodology to measure students' efficiency. Our results show that the divergences detected among regions continue even when information about socioeconomic background, quality of resources and peer effects are taken into account in the analysis.

The influence of exogenous variables on student efficiency shows that the teacher-student ratio is not a significant variable for explaining students' efficiency results. This result entails strong implications for the educational policies implemented by many Spanish regional governments to reduce class size in schools. Moreover, the type of school does not seem to have an influence on the results ei-

(18) The interpretation of elasticities refers to the mean values because the original variables were transformed into deviations from to the mean values.

ther because, after considering the socioeconomic characteristics of school students, private and goverment dependent schools obtain similar results to public ones.

In contrast, students repeating courses or those who were born in a foreign country have worse results in terms of efficiency. These results reveal the need to implement specific policies aimed at improving the academic performance of these students, including hiring support teachers, improving teachers' training to cater for diversity and strengthening the role of social workers when it comes to making parents aware of the importance of education. Likewise, school size or belonging to a particular region, with the exception of Andalusia, Catalonia and the rest of Spain, have a positive effect on the results, La Rioja and Castile-Leon having the most efficient educational systems in Spain.

Furthermore, an important advantage of our study is the interpretation of output and input elasticities. After carrying out this analysis, the results show that the output-input elasticities are positive in all cases, although the school resources' impact on the students' scores in closed to zero, which shows there is no influence of improving scholar resources over the mean students' achievement. On the opposite, the socio-economic background and the peer-group effect present both of them a positive and significant influence on the students' scores. This result claims for a deep revision of the actual system of assigning students into the public-financed schools which is strongly based on the proximity to residence and socio-economic level criteria.

To sum up, we consider our results have relevant implications for regional educational policy, whose guidelines should be focus on enhancing the students' effort and, even more, taking into account the scarce percentage of students' result variance explained by the schools. Nevertheless, these conclusions should be interpreted with caution since they are referred to cross-sectional data from a single year.

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RESUMEN

La reciente publicación del Informe PISA 2006 pone de manifiesto diferencias en el rendimiento educativo de los alumnos procedentes de las diferentes regiones españolas participantes en dicho proyecto. El objetivo de este artículo consiste en identificar las posibles causas de estas divergencias una vez que controlemos el efecto de los inputs educativos y de las variables ambientales. Para ello se estima una función distancia estocástica, que nos permite incorporar un proceso educativo *multi-input* y *multi-output* sujeto a comportamientos ineficientes a nivel del alumno. Los resultados sugieren que La Rioja y Castilla-León son las regiones más eficientes, mientras que, por el contrario, Andalucía, Cataluña y el grupo formado por regiones sin muestra representativa en PISA son las menos eficientes. No obstante, la mayor parte de la divergencia en eficiencia se atribuye a los alumnos con determinadas características: inmigrantes y repetidores, fundamentalmente. Por otro lado, el tamaño de la clase o la titularidad de la escuela no parecen determinar el resultado académico de los alumnos.

Palabras clave: educación, eficiencia, función distancia.

Clasificación JEL: C14, H52, I21.